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**MODES WITH COMPLEX PROPAGATION CONSTANT IN RECTANGULAR GUIDES LOADED  
WITH TRANSVERSELY MAGNETIZED LOSSLESS FERRITE**

by

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**Summary.**

The complete modal spectrum in the complex plane of the propagation constant for a rectangular guide partially filled with transversely magnetized lossless ferrite has been investigated and some numerical calculations have been carried out.

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In a previous work<sup>(1)</sup> we have investigated the modal spectrum for a rectangular guide with perfectly conducting walls partially filled with transversely magnetized lossless ferrite (Figure 1).

Our discussion was however limited to real values of the propagation constant  $k_x$ .

It is the purpose of this work to investigate the modal spectrum in the whole complex  $k_x$  plane. Our discussion will be limited to zero order modes ( $k_z = 0$ ).

The characteristic equation to be investigated has been known for some time<sup>(2)</sup>, and it is the following.

$$(1) \quad \left( \frac{\mu_2}{\mu_1} k_x \sin k_{y2} b_f + k_{y2} \cos k_{y2} b_f \right) \sin k_{y0} b_0 + k_{y0} \frac{\mu_1^2 - \mu_2^2}{\mu_1} \sin k_{y2} b_f \cos k_{y0} b_0 = 0.$$

In equation (1) dependence of the form  $\exp[j(\omega t - k_x x)]$  has been assumed, the propagation constants  $k_x$ ,  $k_{y0}$  and  $k_{y2}$  are measured by assuming as unit  $\omega\sqrt{\mu_0 \epsilon_0}$ , and

$$k_{y0} = \sqrt{1 - k_x^2} \quad ; \quad k_{y2} = \sqrt{\epsilon - k_x^2} \quad ; \quad \epsilon^2 = \epsilon \frac{\mu_1^2 - \mu_2^2}{\mu_1},$$

where  $\epsilon$  is the relative dielectric constant of the ferrite and  $\mu_1$  and  $\mu_2$  are the components of the ferrite tensor relative permeability:

$$\mu = \begin{vmatrix} \mu_1 & j\mu_2 & 0 \\ -j\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

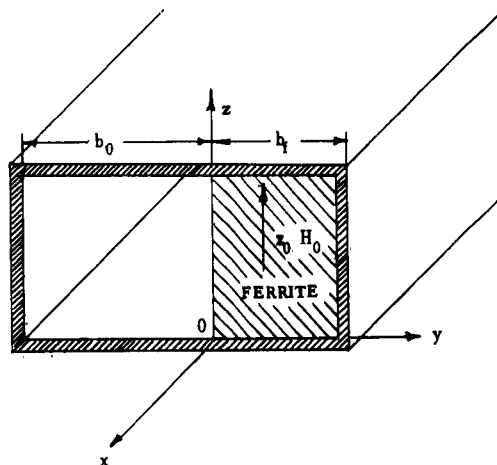


Fig. 1 - The rectangular guide partially filled with ferrite.

(1) - G. Barzilai and G. Gerosa, "Modes in Rectangular Guides Partially Filled with Transversely Magnetized Ferrite," IRE Trans. on Antennas and Propagation, vol. AP-7, Special Supplement, pp. 547-549, December 1959. For details: Istituto Elettrotecnico dell'Università di Roma - Techn. Note No. 1, Contract No. AF 61(052)-101, June 3, 1959.

(2) - M.L. Kales, H.N. Chait and N.G. Saklode, "A Nonreciprocal Microwave Component," J. Appl. Phys., vol. 24, pp. 816-817, June 1953 - errata: p. 1528, December 1953.

For a lossless ferrite  $\mu_1$  and  $\mu_2$  are real and are given by the following expressions :

$$\mu_1 = 1 + \frac{\rho}{1 - \tau^2} \quad ; \quad \mu_2 = \frac{\tau \rho}{1 - \tau^2} \quad ,$$

where :

$$\rho = \frac{M_0}{\mu_0 H_0} \quad ; \quad \tau = \frac{\omega}{\omega_0} \quad ; \quad \omega_0 = -\gamma H_0 \quad ;$$

$M_0$  is the saturation magnetization of the ferrite,  $H_0$  is the applied d. c. magnetic field,  $\gamma$  is the gyromagnetic ratio for the electron.

To our knowledge discussions of equation (1) have been rather limited and the only numerical results available for lossless structures refer to real values of the propagation constant.

In what follows we shall consider only lossless structures. It is easy to verify that if  $k_x$  is a solution of (1),  $k_x^*$  (the asterisk indicates the complex conjugate) is also a solution.

To simplify our analysis we shall assume  $b_f = b_0$ . For this case it is easy to carry out an asymptotic analysis by assuming :

$$(2) \quad |k_x| > 1 \quad \text{and} \quad |k_x| > |k_2^2|$$

and in addition :

$$\mu_1 + \mu_2 \neq 0 \quad \text{and} \quad \mu_1 - \mu_2 + 1 \neq 0 .$$

From (1) we obtain the following asymptotic solutions ( $k_x = k_x^R + j k_x^J$ ) :

$$(3) \quad \begin{cases} k_x^R = 0 \\ k_x^J = \frac{n \pi}{b_f} \end{cases}$$

and

$$(4) \quad \begin{cases} k_x^R = \frac{1}{b_f} \tanh^{-1} \frac{\mu_2^2 - \mu_1^2 - \mu_1}{\mu_2} \\ k_x^J = \frac{n \pi}{b_f} \end{cases}$$

provided that :

$$0 < \frac{\mu_2^2 - \mu_1^2 - \mu_1}{\mu_2} < 1$$

For a lossless ferrite  $\mu_1$  and  $\mu_2$  are real and are given by the following expressions :

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where :

$$\rho = \frac{M_0}{\mu_0 H_0} ; \quad \tau = \frac{\omega}{\omega_0} ; \quad \omega_0 = -\gamma' H_0 ;$$

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To simplify our analysis we shall assume  $b_f = b_0$ . For this case it is easy to carry out an asymptotic analysis by assuming :

$$(2) \quad |k_x| \gg 1 \quad \text{and} \quad |k_x| \gg |t_2^2|$$

and in addition :

$$\mu_1 + \mu_2 \neq 0 \quad \text{and} \quad \mu_1 - \mu_2 + 1 \neq 0 .$$

From (1) we obtain the following asymptotic solutions ( $k_x = k_x^R + j k_x^J$ ) :

$$(3) \quad \begin{cases} k_x^R = 0 \\ k_x^J = \frac{n \pi}{b_f} \end{cases}$$

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provided that :

$$0 < \frac{\mu_2^2 - \mu_1^2 - \mu_1}{\mu_2} < 1$$

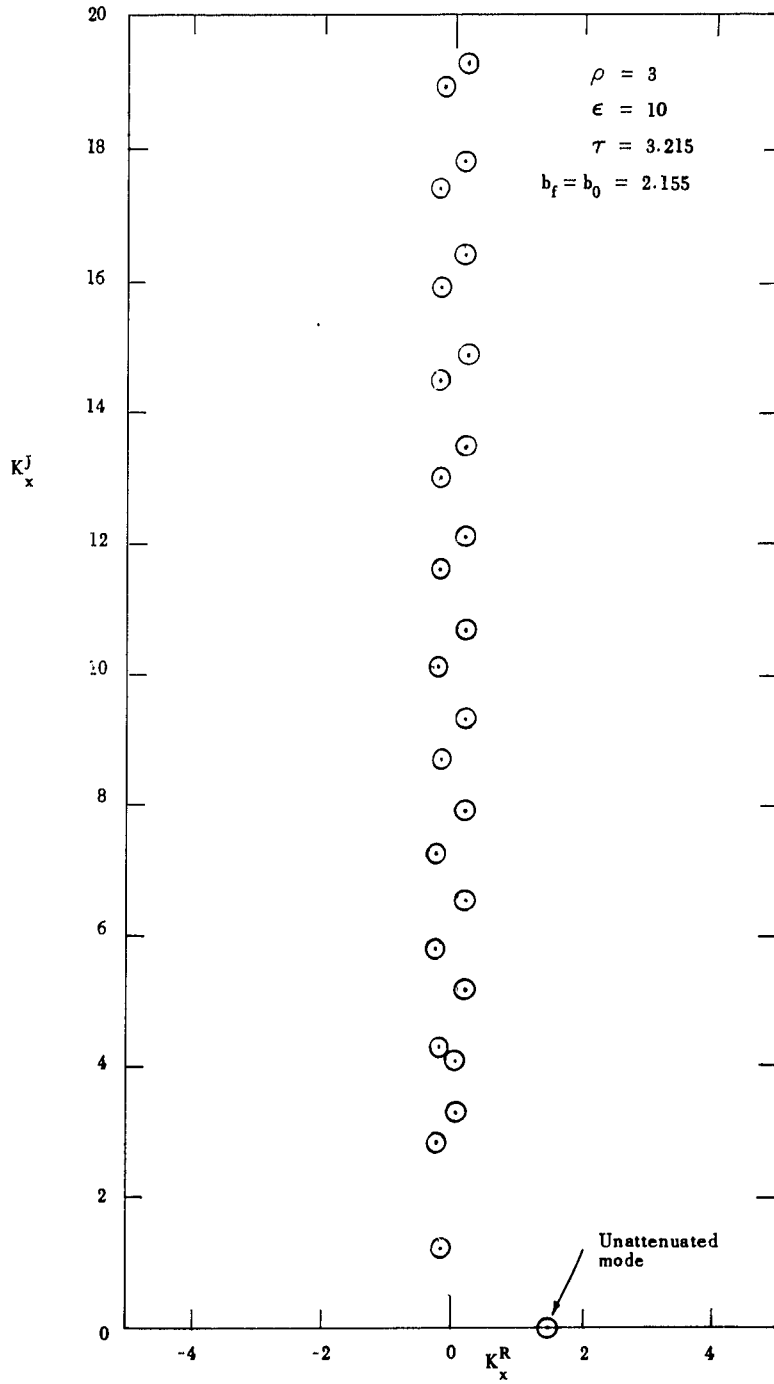


Fig. 2 - Solutions of equation (1) in the complex plane of the propagation constant  $k_x = k_x^R + j k_x^J$ . The numerical results refer to the numerical values of  $\rho$ ,  $\epsilon$ ,  $\tau$ ,  $b_f$  and  $b_0$  indicated in the figure. These values may be taken to correspond to

$$M_0 = 0.3 \text{ Wb/m}^2 \quad H_0 = 10^6 / 4\pi \text{ A/m} \quad ; \quad f = \omega / 2\pi = 9000 \text{ Mc/s} \quad ; \quad (b_f + b_0) / \omega \sqrt{\mu_0 \epsilon_0} = 0.9''.$$

or :

$$(5) \quad \begin{cases} k^R = \frac{1}{b_f} \coth^{-1} \frac{\mu_2^2 - \mu_1^2 - \mu_1}{\mu_2} \\ k_x^J = \frac{(n + 1/2) \pi}{b_f} \end{cases}$$

if :  $\frac{\mu_2^2 - \mu_1^2 - \mu_1}{\mu_2} > 1$  .

In (3), (4) and (5)  $n$  is any integral number sufficient to fulfill (2).

We have considered a numerical case for which we have solved equation (1) without any asymptotic approximation. The values of the parameters are indicated in Fig. 2, in which the solutions of equation (1) are recorded only in the  $k_x^J \geq 0$  plane, since the solutions are symmetric with respect to the real axis.

The solution of equation (1) have been obtained by separating real and imaginary parts and setting them separately equal to zero. In the plane  $k_x^R, k_x^J$ , each of these two equations is represented by a curve with many branches. From the intersections of these two curves we obtain the solutions sought.

We have investigated the following zone of the  $k_x$  complex plane :

$$-20 \leq k_x^R \leq 20 \quad ; \quad 0 \leq k_x^J \leq 20$$

This investigation has required about twenty five hours of a medium speed electronic digital computer.

With the numerical values assumed in Fig. 2, the asymptotic solutions are given by (3) and (4) .

From the numerical results and the asymptotic analysis it appears that the structure considered possesses only one zero order unattenuated propagating mode and an infinite number of attenuated propagating modes, lying in a narrow strip including the imaginary axis.

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